Chapter 3 ACTIVE FILTERS

1. HISTORICAL

Electrical filters, invented by Zobel as early as 1923, have significantly expanded telecommunications capabilities. Until recent years, they were predominantly realized using passive components with resonant properties such as inductors, capacitors, quartz… etc. The advent of the transistor, and more recently, the integrated operational amplifier, has enabled the construction of a new type of resonator using only resistors and capacitors associated with these active elements. Active filters offer numerous advantages, particularly in the low-frequency domain. They are lightweight, compact, and inexpensive.

2. Definition

If the operating frequency of a circuit is varied, the impedance of capacitors and inductors will also vary. It is important to understand the behavior of these elements as the frequency changes. We will see that a judicious choice of these components will allow the creation of circuits where signals of certain desired frequencies can be blocked or allowed to pass. This type of circuit is called a filter.

The frequency filtering function serves to suppress unwanted frequency signals and preserve or even amplify desired frequency signals.

A filter is a passive quadripole (or a combination of passive quadripoles) that is inserted between a source and a load impedance. It allows signals of certain sinusoidal frequencies to pass without attenuation, while completely blocking signals of other frequencies.

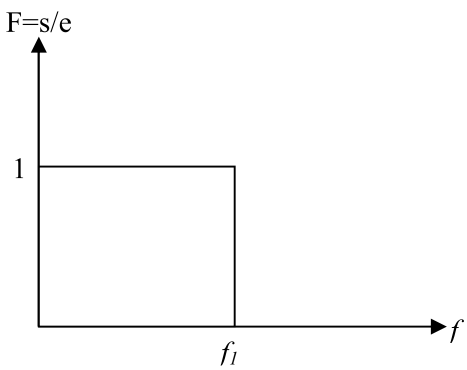
Filters come in various forms. When there is no amplification of the input signal power by an active component (transistor, operational amplifier), it is passive; otherwise, it is active. Active filters are composed solely of resistors, capacitors, and active elements (usually operational amplifiers).

3. Action of Different Filters

According to the frequency domain eliminated, filters are classified into four categories: low-pass, high-pass, band-pass, and band-stop (or notch) filters.

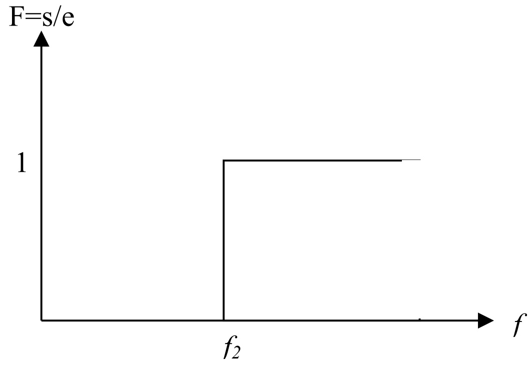
- **Low-pass filter**

It allows signals with frequencies $0 < f < f1$ to pass without attenuation and attenuates frequencies from $f1$ ***to*** $f\infty$.



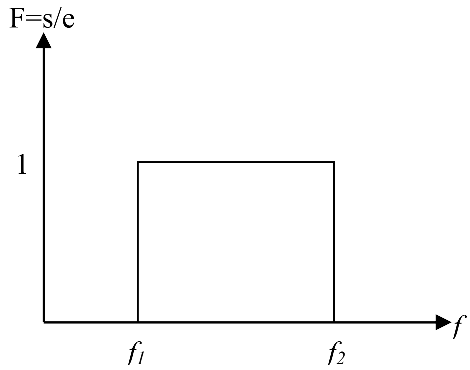
- **High-pass filter**

It allows signals with frequencies ***f > f2*** to pass without attenuation and attenuates frequencies ***f < f2***.



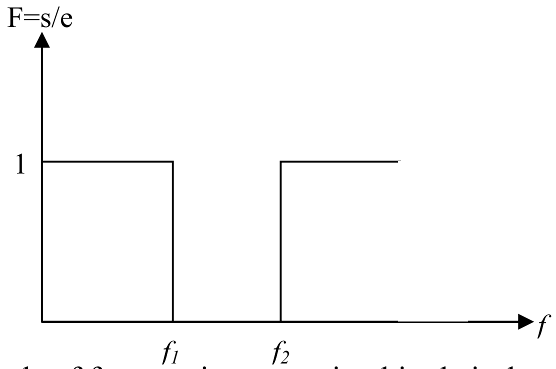
- **Band-pass filter**

It allows signals with frequencies ***f1 < f < f2*** to pass without attenuation and attenuates other frequencies.



- **Band-stop filter**

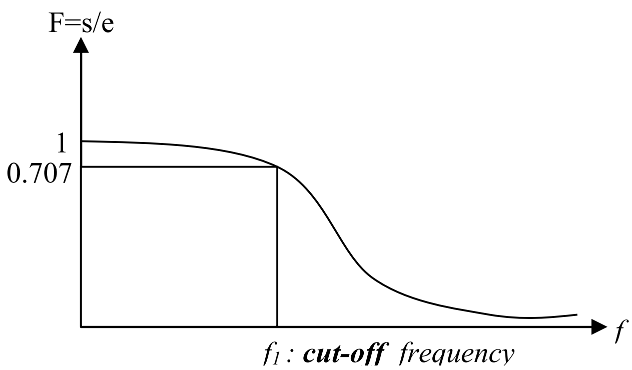
It allows signals with frequencies not between ***f1*** and ***f2*** to pass without attenuation.



In reality the filters transmit more or less well the signals of frequencies transmitted in their domains of definitions, and attenuate more or less well the others.

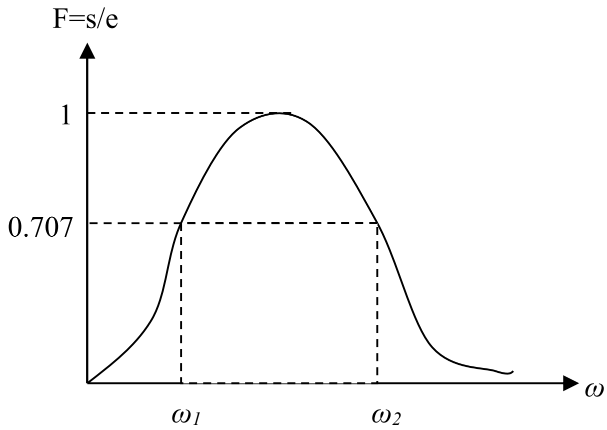
- **Example :**

The gain curve of a real low-pass filter is:



We then consider that the frequency ***f1*** called **cut-off** frequency is the frequency for which the attenuation is ***-3dB***

$$\frac{1}{\sqrt{2}} = 0.707; 20\log(0.707) = -3dB$$



4. Characteristics of a Filter

The main characteristics of an active filter are:

- Its cutoff frequency or frequencies,
- Its bandwidth (for band-pass and band-stop filters),
- Its maximum voltage amplification coefficient and maximum gain.

These characteristics, in a first approximation, depend only on the passive components used.

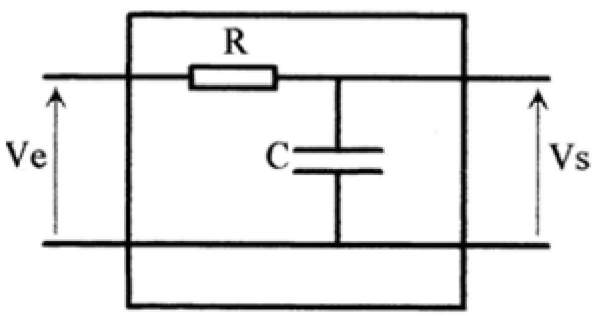
5. Structure of Active Filters

There are a large number of configurations for implementing active filters. We will mention a few typical structures that are encountered very frequently.

6. Classic First-Order Filters

According to the frequency domain transmitted without attenuation, we distinguish:

6.1. Low-pass filter



The transfer function is expressed in the form:

$$F(j\omega) = \frac{k}{1 + j\frac{\omega}{\omega_0}}$$

where k is a real constant and ω_0 is the cutoff frequency.

6.1.1. Bode Plot

$$\frac{V_s}{V_e} = \frac{1}{1 + jRC\omega}$$

General form

$$\frac{V_s}{V_e} = \frac{k}{1 + j\frac{\omega}{\omega_0}} = F$$

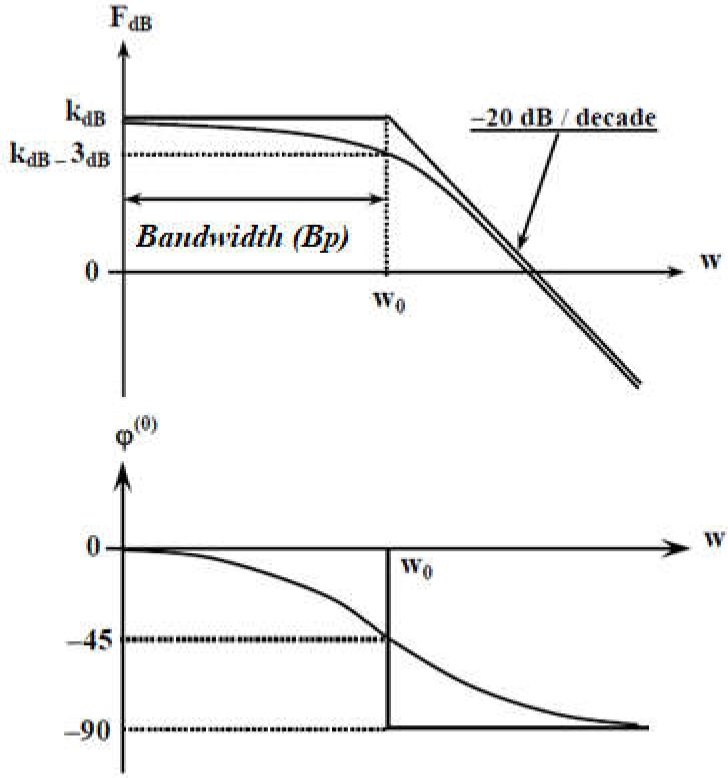
$$k = 1, \omega_0 = \frac{1}{RC}$$

$$F = \frac{k}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

$$\Rightarrow F_{dB} = 20logF$$

$$F_{dB} = 20\ logk - 20\ log\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}$$

$$\varphi = -arctg\left(\frac{\omega}{\omega_0}\right)$$



Gain and Phase Curves

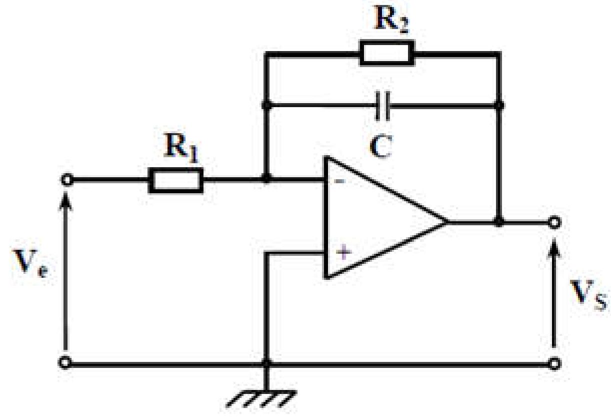
$$\omega \ll \omega_0 \Rightarrow \begin{cases} F_{dB} \to k_{dB} \\ \varphi \to 0 \end{cases}$$

$$\omega = \omega_0 \Rightarrow \begin{cases} F_{dB} \to k_{dB} - 3dB \\ \varphi \to -\frac{\pi}{4} \end{cases}$$

$$\omega \gg \omega_0 \Rightarrow \begin{cases} F_{dB} \to k_{dB} - 20\ log\left(\frac{\omega}{\omega_0}\right) \\ \varphi \to -\frac{\pi}{2} \end{cases}$$

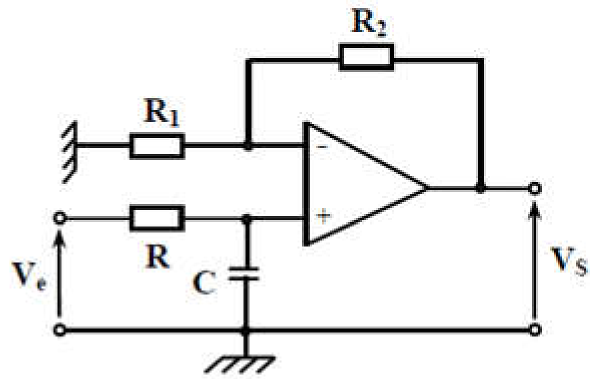
6.1.2. Examples

$$\begin{cases} V^{+} = 0 \\ V^{-} = \dfrac{R_1 V_s + (R_2 // C) V_e}{R_1 + (R_2 // C)} \end{cases} \Rightarrow F(p) = \frac{-\left(\dfrac{R_2}{R_1}\right)}{1 + R_2 C p}$$



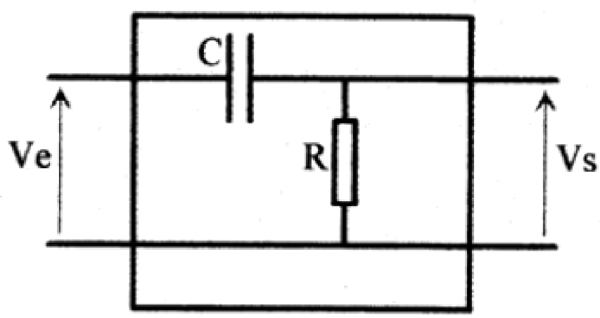
Low-pass filter

$$\begin{cases} V^{-} = \dfrac{R_1}{R_1 + R_2} V_s \\ V^{+} = \dfrac{1}{1 + RCp} V_e \end{cases} \Rightarrow F(p) = \frac{\left(1 + \dfrac{R_2}{R_1}\right)}{1 + RCp}, \ V^{+} = V^{-}$$



Low-pass filter

6.2. High-pass Filter

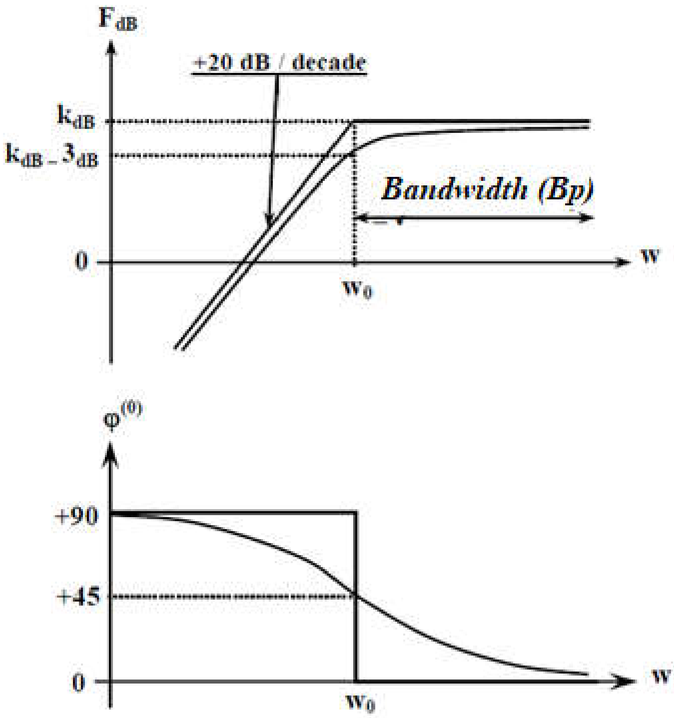


The transfer function is expressed in the form::

$$F(j\omega) = k \frac{j \dfrac{\omega}{\omega_0}}{1 + j \dfrac{\omega}{\omega_0}}$$

where ***k*** is a real constant and ω_0 is the cutoff frequency.

6.2.1. Bode Plot



Gain and Phase Curves

$$\frac{V_s}{V_e} = \frac{jRC\omega}{1 + jRC\omega}$$

General form

$$\frac{V_s}{V_e} = \frac{k\,\frac{\omega}{\omega_0}}{1 + j\frac{\omega}{\omega_0}} = F$$

$$k = 1, \omega_0 = \frac{1}{RC}$$

$$F = k\frac{\frac{\omega}{\omega_0}}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}} \Rightarrow F_{dB} = 20logF$$

$$F_{dB} = k_{dB} + 20\;log\left(\frac{\omega}{\omega_0}\right) - 20\;log\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}$$

$$\varphi = \frac{\pi}{2} - arctg\left(\frac{\omega}{\omega_0}\right)$$

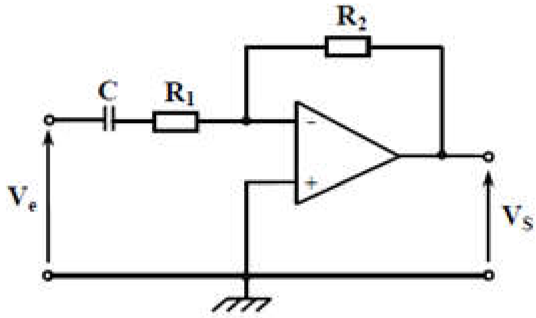
$$\omega \ll \omega_0 \Rightarrow \begin{cases} F_{dB} \rightarrow -\infty \\ \varphi \rightarrow \frac{\pi}{2} \end{cases}$$

$$\omega = \omega_0 \Rightarrow \begin{cases} F_{dB} \rightarrow k_{dB} - 3dB \\ \varphi \rightarrow \frac{\pi}{4} \end{cases}$$

$$\omega \gg \omega_0 \Rightarrow \begin{cases} F_{dB} \rightarrow 20\log k = k_{dB} \\ \varphi \rightarrow 0 \end{cases}$$

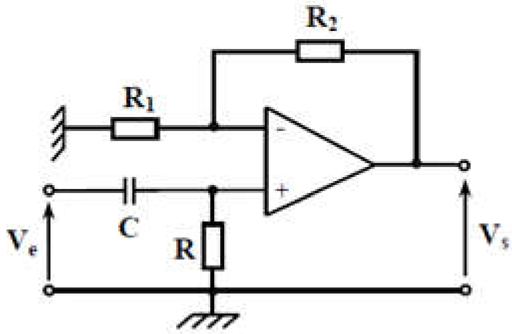
6.2.2. Examples

$$\begin{cases} V^+ = 0 \\ V^- = \dfrac{R_2 V_e + \left(R_1 + \dfrac{1}{Cp}\right) V_s}{R_1 + R_2 + \dfrac{1}{Cp}} \end{cases} \Rightarrow F(p) = -\left(\frac{R_2}{R_1}\right)\left(\frac{R_1 C p}{1 + R_1 C p}\right)$$



High-pass filter

$$\begin{cases} V^- = \dfrac{R_1}{R_1 + R_2} V_s \\ V^+ = \dfrac{RCp}{1 + RCp} V_e \end{cases} \Rightarrow F(p) = \left(1 + \frac{R_2}{R_1}\right)\frac{RCp}{1 + RCp}$$



High-pass filter